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TRANSIENT PLANETARY WAVES IN SEMI-BOUNDED CHANNELS EXTENDING ALONG A MERIDIAN†

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The unsteady-state boundary-value problem of the propagation of planetary waves in semi-bounded channels running north–south is studied in the β-plane approximation. An explicit solution is obtained and the behaviour of normal transient waves at long times is investigated. © 1998 Elsevier Science Ltd. All rights reserved.

Planetary waves, or Rossby waves are low-frequency oscillations in the ocean and the atmosphere, caused by the action on the moving medium of the Coriolis force due to the diurnal rotation of the Earth about its axis. In many cases transient planetary waves are aptly described by a third-order equation [1, 2] in the β-plane approximation, in which the spherical surface of the Earth is replaced locally by a tangent plane.

The solvability of initial-boundary-value problems for pseudo-parabolic evolution equations in one space variable has been studied [3] using abstract functional methods. In more than one dimension, asymptotic formulae have been obtained in the Cauchy problem and the first boundary-value problem for pseudo-parabolic equations in a quadrant [4, 5]. Various properties of both linear and non-linear planetary waves have been studied [2, 6], including, in particular, problems of instability. The influence of the relief on the propagation of planetary waves, as well as their reflection in lakes and semi-bounded channels with an uneven bottom, have been investigated in considerable detail by numerical methods [7–11].

This paper is a sequel to [12] from a mathematical point of view. Here, however, unlike [12], an explicit solution is constructed for the initial-boundary-value problem of the formation of transient planetary waves in a semi-infinite channel along a meridian and not along a parallel as in [12].

In addition, while the problem in [12] was solved by classical separation of variables, the problem studied here is not tackled directly by that method but by a modification, proposed here, for the method of separation of variables.

As in [12], the asymptotic behaviour of the solution at long times will be considered. A comparative analysis of the results with those of [12] is also presented.

Planetary waves have received practically no attention in the transient state, since they are described by a non-classical third-order equation. It was not clear previously what type of perturbations is described by this equation, whether its solutions are really wave-like or are like the solutions of the parabolic equations that describe heat transfer, not possessing a wave-like structure. One of the main results of this paper is a detailed description of the pattern of transient planetary waves. It turns out that, unlike acoustic or electromagnetic waves, which are described by hyperbolic equations and have a front propagating at a finite speed, transient planetary waves have a quasi-front. At points where the quasi-front has already passed, the perturbations have a wave-like mode. At points where the quasi-front has already passed, the perturbations have a wave-like mode. At points not yet reached by the quasi-front they represent a process of exponential decay; they do not describe waves but resemble solutions of parabolic equations.

Thus, the equation of planetary waves, on the one hand, describes long-range interactions, as is characteristic for heat transfer and parabolic equations, and on the other hand, it describes wave phenomena characteristic of acoustic and electromagnetic waves governed by hyperbolic equations.

Note that the existence of solutions has been proved [3–5] for certain initial-boundary-value problems for equations similar to the planetary wave equation, and estimates have been obtained for the rate of decay of these solutions at long times. However, the fine structure of the transient waves, which is described in this paper, was not considered in previous investigations.

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1. FORMULATION OF THE PROBLEM

The propagation of planetary waves (Rossby waves) along the Earth's surface, when the latter is replaced locally by a tangent plane, is described by the linearized equation [1, 2]

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\alpha^2 u\right)+\beta\frac{\partial u}{\partial x}=0 \tag{1.1}$$

where the x and y axes of the system of coordinates point respectively east (west) and north (south), $u(x, y, t)$ is the stream function, and α and β are known constants. In particular, $\beta = \partial f/\partial y$, where $f = 2\Omega\sin\varphi$ is the Coriolis parameter, Ω is the angular velocity of the Earth's rotation, and φ is the latitude of the position, taken with a plus sign in the northern hemisphere and a minus sign in the southern hemisphere. In this notation, $f > 0$ in the northern hemisphere and $f < 0$ in the southern hemisphere. In barotropic oscillations of a liquid layer of depth H it may be assumed that $\alpha^2 = f^2/(gH)$, where g is the acceleration due to gravity.

We transform Eq. (1.1) to dimensionless variables $\alpha\ \mathrm{sign}(\alpha\beta)x$, $\alpha\ \mathrm{sign}(\alpha\beta)y$, $|\beta/\alpha|t$, retaining the previous notation for the new variables. Then (1.1) becomes

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-u\right)+\frac{\partial u}{\partial x}=0 \tag{1.2}$$

Suppose that the liquid occupies a two-dimensional semi-bounded channel-waveguide: $Q = \{(x, y) \in R^2: 0 < x < \pi, 0 < y < +\infty\}$. Let $C^{(1)}([0, +\infty); \mathring{W}_2^1(0, \pi))$ denote the class of continuously differentiable abstract functions of t with values in the Banach space $\mathring{W}_2^1(0, \pi)$, and let $C_0^{(1)}([0, +\infty); \mathring{W}_2^1(0, \pi))$ be the class of abstract functions that belong to $C^{(1)}([0, +\infty); \mathring{W}_2^1(0, \pi))$ and vanish at $t = 0$.

We will say that a function $V(x, y, t)$, defined in $\overline{Q} \times [0, +\infty)$, belongs to class M_γ if a number $\gamma > 0$ and a function $c(t) \in C^0[0, +\infty)$ exist such that $|V(x, y, t)| \leqslant c(t)\exp(-\gamma y)$ and $y \geqslant 1, t \geqslant 0, x \in [0, \pi]$.

We will now formulate the main problem to be considered below.

Problem A. Find a function $u(x, y, t)$, continuous in $\overline{Q} \times [0, +\infty)$, which satisfies Eq. (1.2) in the classical sense in $Q \times (0, +\infty)$, and the following conditions

$$u|_{t=0} = 0, \quad u|_{x=0} = u|_{x=\pi} = 0 \tag{1.3}$$

$$u|_{y=0} = F(x,t) \in C_0^{(1)}([0,+\infty); \ \mathring{W}_2^1(0,\pi)) \tag{1.4}$$

$$\frac{\partial^k}{\partial y^k}\frac{\partial^p}{\partial t^p}u \in M_\gamma, \quad k,p=0,1 \tag{1.5}$$

All the conditions of the problem are satisfied in the classical sense.

By solving this problem we can study, for example, the nature of planetary waves in a long bay stretching along a meridian, if one end of the bay opens into the ocean. The function $F(x, t)$ represents the action of the ocean on that end of the bay. The generation of waves begins at a time $t = 0$. Impermeability conditions are specified on the side walls ($x = 0, x = \pi$).

Remark. If the channel extends into the lower half-plane, the solution there is given by a function $u(x, -y, t)$, where $u(x, y, t)$ is the solution of Problem A for a channel in the upper half-plane.

We will say that a solution $u(x, y, t)$ of Problem A belongs to smoothness class $\mathcal{K}$ if:

1. the derivatives

$$\frac{\partial^k}{\partial x^k}\frac{\partial^p}{\partial t^p}u$$

for $k, p = 0, 1; x \in [0, \pi], y > 0, t > 0$, are continuous in all their arguments;

2. the function $y(x, y, t)$ has classical derivatives

$$\frac{\partial^k}{\partial y^k}\frac{\partial^p}{\partial t^p}u$$

in $Q \times (0, \infty)$ $(k, p = 0, 1)$ which, as abstract functions of y and t with values in $L_2(0, \pi)$, are continuous for $(y, t) \in [0, \infty) \times [0, T]$ for any $T > 0$.

The last conditions should be viewed as conditions of smooth approach to the boundaries $x = 0$, $x = \pi$, $t = 0$.

Theorem 1. Problem A has a unique solution in the smoothness class $\mathcal{K}$.

The proof is based on the energy identify for Eq. (1.2), which in the present case has the form

$$\frac{d}{dt}(\|\nabla u\|^2_{L_2(Q)} + \|u\|^2_{L_2(Q)}) = -2(u, u_{ty})_{L_2(0,\pi)}\Big|_{y=0}$$

2. CONSTRUCTION OF THE SOLUTION

Problem A, unlike the analogous problem for a channel along the x axis (see [12]), cannot be solved directly by separation of variables. It is our object to reduce Problem A to a certain auxiliary problem which can be solved by that method. To do this we consider the following auxiliary problem.

Problem B. Find a function $U(x, y, t)$, continuous in $\overline{Q} \times [0, \infty)$, which satisfies the following integro-differential equation in $Q \times (0, +\infty)$, in the classical sense

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - U\right) - \frac{1}{4}\int_0^t U(x, y, \tau)d\tau = 0$$

and the following conditions

$$U|_{t=0} = 0, \quad U|_{x=0} = U|_{x=\pi} = 0 \tag{2.1}$$

$$U|_{y=0} = \mathcal{F}(x, t) \in C_0^{(1)}([0, +\infty);\ \overset{\circ}{W}{}_2^1(0, \pi)) \tag{2.2}$$

$$\frac{\partial^k}{\partial y^k}\frac{\partial^p}{\partial t^p} U \in M_\gamma, \quad k, p = 0, 1 \tag{2.3}$$

All the conditions of the problem are satisfied in the classical sense.

Let us introduce the following functions

$$I(x, t) = \sqrt{\frac{x}{2t}} I_1(\sqrt{2xt}), \quad J(x, t) = \sqrt{\frac{x}{2t}} J_1(\sqrt{2xt}) \tag{2.4}$$

where $J_1(\xi)$, $I_1(\xi)$ are first-order Bessel and Infield functions, respectively.

The relation between Problem B and Problem A is revealed by the following lemma.

Lemma 1. Suppose that the function $\mathcal{F}(x, t)$ in boundary condition (2.1) of Problem B is related to the function $F(x, t)$ in boundary condition (1.4) of Problem A as follows:

$$\mathcal{F}(x, t) = F(x, t) + \int_0^t I(x, t - \tau)F(x, \tau)d\tau$$

In that case, if $U(x, y, t)$ is a solution of Problem B in class $\mathcal{K}$, the function $u(x, y, t)$ defined by

$$u(x, y, t) = U(x, y, t) - \int_0^t J(x, t - \tau)U(x, y, \tau)d\tau$$

is a solution of Problem A in class $\mathcal{K}$.

The proof consists of a direct check.

Thus, in order to solve Problem A, it will suffice to find a solution of Problem B, and that can be done by the classified method of separation of variables [12]. With that accomplished, the solution of Problem B in class $\mathcal{K}$ is expressed as a series in the functions $\{\sqrt{(2/\pi)} \sin nx\}_{n=1}^{\infty}$, which form a complete orthonormal system in $L_2(0, \pi)$; the solution is given by the following formula

$$U(x,y,t) = \sum_{n=1}^{\infty} \sin nx \ \ \varphi_n(y,t)$$

where

$$\varphi_n(y,t) = \frac{2}{\pi} \nu_n(t) \exp(-y\sqrt{n^2+1}) - \frac{1}{\sqrt{n^2+1}} \int_0^t B_n(y, t-\tau) \nu_n(\tau) d\tau \tag{2.5}$$

$$\nu_n(t) = \int_0^{\pi} \sin ns \ \ \mathcal{F}(s,t) ds \tag{2.6}$$

$$B_n(y,t) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{\mu \exp(i\mu y\sqrt{n^2+1})}{(\mu^2+1)^{3/2}} \sin\left(\frac{t}{2\sqrt{\mu^2+1}\sqrt{n^2+1}} \right) d\mu \tag{2.7}$$

which may be verified by a direct check.

Using Lemma 1, we will construct a solution of Problem A.

Theorem 2. A solution of Problem A in the smoothness class $\mathcal{K}$ exists, is unique and has the form

$$u = \sum_{n=1}^{\infty} \sin nx \ \ u_n(x,y,t) \tag{2.8}$$

$$u_n(x,y,t) = \frac{2}{\pi} (\varphi_n(y,t) - \int_0^t J(x, t-\tau) \varphi_n(y,\tau) d\tau)$$

The functions $\varphi_n(y, t)$ are defined by formula (2.5) with

$$\nu_n(t) = \int_0^{\pi} \sin ns \left[F(s,t) + \int_0^t I(s, t-\tau) F(s,\tau) d\tau \right] ds \tag{2.9}$$

where we have used the notation of (2.4) and (2.7).

We have thus constructed a solution of Problem A in explicit form. We will now investigate the solution.

3. THE ASYMPTOTIC BEHAVIOUR OF THE SOLUTION AT LONG TIMES

To study the asymptotic behaviour of the functions $u_n(x, y, t)$ introduced in (2.8), which will be referred to as "normal waves," as $t \to +\infty$, we will derive a different representation, using Laplace transforms with respect to t. We will assume that wave propagation is produced in the channel by an excitation which is finite in time, that is, in boundary condition (1.4), $F(x, t) \equiv 0$ for $t > T$ for some $T > 0$.

Starting now from formulae (2.5)–(2.9) and using Laplace transforms, we obtain a new integral representation for the normal waves

$$u_n(x,y,t) = \frac{2}{\pi} \int_0^{\pi} \sin ns \ \ G_n(x,s,y,t) ds$$

$$G_n(x,s,y,t) = \frac{2}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \exp\left(tp - \frac{x-s}{2p} - y\sqrt{n^2+1+\frac{1}{4p^2}} \right) \hat{F}(s,p) dp \tag{3.1}$$

where $\hat{F}(s, p)$ is the Laplace transform of $F(s, t)$, $\sigma > 0$.

To get an idea of the nature of the normal waves propagating in the channel, we set $y = \theta t$ and study the asymptotic behaviour of integral (3.1) for fixed θ and $t \to +\infty$, using the method of steepest descent. The saddle-points in this case are the roots of the equation

$$dS/dp = 0 \quad \left(S(p) = p - \theta\sqrt{n^2 + 1 + \frac{1}{4p^2}} \right) \tag{3.2}$$

We shall assume below that, in order to single out a single-valued branch of the function S, the complex p plane is cut along the segment of the imaginary axis between the branch points $p_{1,2} = \pm i(2\sqrt{(n^2+1)})^{-1}$ that contains the point $p = 0$.

We define a new complex variable $z = (2p\sqrt{(n^2+1)})^{-1}$ and a parameter

$$k = (2\sqrt{3}\ \theta(n^2+1))^{-1} \tag{3.3}$$

and rewrite Eq. (3.2) as follows:

$$-z^3 = k\sqrt{3}\sqrt{1+z^2} \tag{3.4}$$

It can be seen that the roots of Eq. (3.2) must be sought among the roots of the sixth-degree algebraic equation

$$z^6 - 3k^2z^2 - 3k^2 = 0 \tag{3.5}$$

which in turn reduces to the following auxiliary third-degree equation in $w = z^2$

$$w^3 - 3k^2w - 3k^2 = 0 \tag{3.6}$$

This equation has three real roots if $k > 3/2$ and one real root if $0 < k < 3/2$.

Thus, the critical value $k^* = 3/2$ enables us to determine, through (3.3), the critical value of the parameter $\theta = y/t$; consequently, the equation of the quasi-front of the normal wave will be $y = (v_f)_n t$, where $(v_f)_n = (3\sqrt{3}(n^2+1))^{-1}$ is the velocity of the quasi-front.

Since the asymptotic representation of integral (3.2) with $y = \theta t$ and $t \to +\infty$ has qualitatively different representations depending on the value of the parameter k, and hence also on the parameter θ, we must consider two cases.

1. Suppose $k > 3/2$. Then $y < (v_f)_n t$. This means that the quasi-front of the nth normal wave has already passed through the point with coordinates (x, y) in the channel Q. The real roots of Eq. (3.6) in this case are

$$w_{1,2} = -2k\cos\left(\eta \pm \frac{\pi}{3}\right),\quad w_3 = 2k\cos\eta,\quad \eta = \eta(k) = \frac{1}{3}\arccos\frac{3}{2k}$$

where $0 < \eta(k) < \pi/6$ for $k > 3/2$. Note that $w_{1,2} = -3/2$, $w_3 = 3$ when $k = 3/2$ and $\lim w_1 = -1$, $\lim w_2 = -\infty$, $\lim w_3 = +\infty$ as $k \to \infty$.

Corresponding to these roots of Eq. (3.6) there are six roots of Eq. (3.5): $z_{1,2} = \pm i|w_1|^{1/2}$, $z_{3,4} = \pm i|w_2|^{1/2}$, $z_{5,6} = \pm w_3^{1/2}$.

We now draw attention to the fact that, corresponding to the cut made above in the complex p plane, there is a cut in the complex z plane, running along the imaginary axis and connecting the points $z = i$ and $z = -i$ through the point at infinity. We may therefore state that the root z_5 is irrelevant for Eq. (3.4). It is also evident that the root z_6 makes only an exponentially small contribution to the asymptotic behaviour of integral (3.1). By deforming the contour of integration in (3.1) and applying the stationary-phase method, we can now take into account the contributions from the points z_j corresponding to the roots $w_{1,2}$ giving the following result.

Theorem 3. Let the function $F(x, t)$ be in (1.3) be finite in time. Then the following asymptotic formula holds for $y = (2\sqrt{3}(n^2+1)k)^{-1}t$, $t \to +\infty$ and $k > 3/2 + \delta$ $(\delta > 0)$

$$u_n(x,y,t) = \left(\frac{k\sqrt{3}}{\pi t\sqrt{n^2+1}}\right)^{1/2} \left\{ \sum_{j=1}^{2} \frac{1}{\sqrt{\Delta_j}} \mathrm{Re}[F_n(w_j)\exp(i(ta(w_j) + xb(w_j) + \right.$$

$$\left. + yh(w_j) + (-1)^j\frac{\pi}{4}))]\right\} + O\left(\frac{1}{t}\right) \tag{3.7}$$

where

$$h(w)=((|w|-1)(n^2+1))^{1/2},\quad \Delta_j = |w_j|^2|2w_j+3|(|w_j|-1)^{-3/2},\quad j=1,2$$

$$F_n(w)=\frac{2}{\pi}\int_0^T \exp(-i\tau a(w))\int_0^\pi \exp(-isb(w))F(s,\tau)\sin nsdsd\tau$$

$$a(w)=(4|w|(n^2+1))^{-1/2},\quad b(w)=(|w|(n^2+1))^{1/2}$$

and we have used the notation previously introduced.

2. Suppose $0 < k < 3/2$. Then $y > (v_f)_n t$. In this case we consider points of the channel $(x, y) \in Q$ which the quasi-front of the nth normal wave has not yet reached.

At such values of k, Eq. (3.6) has two complex-conjugate roots $\tilde{w}_{1,2} = -(A^+ + A^-)/2 \pm i\sqrt{3}(A^+ - A^-)/2$ and one real root $\tilde{w}_3 = A^+ + A^-$. Here

$$A^{\pm}=(3k^2/2\pm k^2\sqrt{9/4-k^2})^{1/3},\quad A^+ > A^- > 0,\quad A^+A^- = k^2.$$

It is clear from these formulae that

$$\tilde{w}_{1,2}=(3k^2)^{1/3}\exp(\pm i2\pi/3)+o(k^{2/3}),\quad \tilde{w}_3=(3k^2)^{1/3}+o(k^{2/3}),\quad k\to\infty$$

Corresponding to the roots $\tilde{w}_j$ of Eq. (3.6) there are six roots of Eq. (3.5), three of which are irrelevant for Eq. (3.4), while one makes only a negligible contribution to the asymptotic form. In using the method of steepest descents for this case, the only significant contributions to the asymptotic behaviour of integral (3.1) come from the two complex-conjugate roots $\tilde{p}_1$ and $\tilde{p}_2$ of $S(p)$, where

$$\tilde{p}_{1,2}=(2\tilde{z}_{1,2}\sqrt{n^2+1})^{-1},\quad \tilde{z}_{1,2}=|\tilde{w}_1|^{1/2}\exp\{\pm i\arg(\tilde{w}_1^{1/2})\}$$

In order to make the square roots of complex numbers used below single-valued, we will adopt the following conventions

$$\frac{\pi}{3}<\arg(\tilde{w}_1^{1/2})<\frac{\pi}{2},\quad 0<\arg[(1+\tilde{w}_1)^{1/2}]<\frac{\pi}{2},\quad 0<\arg(3+2\tilde{w}_1)<\frac{\pi}{2}$$

Theorem 4. Suppose that the function $F(x, t)$ of (1.3) is finite in time. Then the following asymptotic formula holds for $t = 2\sqrt{3}k(n^2 + 1)y$, $y \to +\infty$ and $\delta < k < 3/2 - \delta$ $(0 < \delta < 3/4)$

$$\begin{aligned}
&u_n(x,y,t)=\exp\{\Phi_1(x,y,t,\tilde{w}_1)\}\frac{1}{\sqrt{2\pi y\tilde{\Delta}(n^2+1)^{3/2}}}\times\\
&\times\operatorname{Im}[\tilde{F}_n(\tilde{w}_1)\exp(i\Phi_2(x,y,t,\tilde{w}_1)+i\varepsilon]+O(y^{-3/2})\}\\
&\Phi_1=\operatorname{Re}\Phi,\quad \Phi_2=\operatorname{Im}\Phi,\quad \Phi=\Phi(x,y,t,w)=\\
&=-x\sqrt{n^2+1}w^{1/2}+\frac{t}{2\sqrt{n^2+1}w^{1/2}}-y\sqrt{n^2+1}(1+w)^{1/2}\\
&\tilde{\Delta}=|\tilde{w}_1|^2|2\tilde{w}_1+3|\,|1+\tilde{w}_1|^{-3/2}\\
&\varepsilon=\frac{3}{2}\arg[(1+\tilde{w}_1)^{1/2}]-\frac{1}{2}\arg(2\tilde{w}_1+3)-2\arg(\tilde{w}_1^{1/2})\\
&\tilde{F}_n(w)=\frac{2}{\pi}\int_0^T d\tau\int_0^\pi \exp\{i\Phi_2(-s,0,-\tau,w)\sin ns\; F(s,\tau)ds
\end{aligned}\tag{3.8}$$

Remark. When the conditions of Theorem 3 are satisfied, we can write

$$\Phi_1(x,y,t,w)=\Phi_0(x,t,w)-y\sqrt{n^2+1}C(w),\quad C(w)>C_0>0,\quad C_0=\text{const}$$

Consequently, for any fixed t in the region ahead of the quasi-front, the functions $u_n(x, y, t)$ decrease exponentially as y increases along the channel.

We will now discuss the asymptotic formulae.

Formula (3.7) describes two travelling waves of different phases and different amplitudes, defined in terms of the quantities Δ_j, $F_n(w_j)$ $(j = 1, 2)$. The phase of each wave depends on the x coordinate. For comparison, in a channel extending along a parallel (studied previously in [12]), the phase velocity is independent of the transverse coordinate. In addition, it has been shown [12] that the quasi-front of a normal transient wave propagates to the west and the east at different speeds, equal respectively to $(n^2+1)^{-1}$ and $(n^2+1)^{-1/8}$ in the dimensionless variables introduced above. The northward and southward velocities of propagation of the quasi-front are the same.

In either type of channel, whether along a meridian or along a parallel, the velocity of propagation of the quasi-front of the nth transient normal wave decreases as n increases and is proportional to $1/n^2$. Hence the maximum velocity of propagation of the quasi-front in either case is that of the wave with $n = 1$. The pattern of transient normal waves in different channels also proves to be similar. In all cases, and at each fixed instant of time, there is a "forerunner", which decreases exponentially along the channel, ahead of the quasi-front. Behind the quasi-front there is a trail of oscillations. At each fixed point of the channel these oscillations decay with time as $1/\sqrt{t}$, as the quasi-front increases its distance from this point. Thus, the perturbations are wavelike only in that part of the channel where the quasi-front of the transient normal wave of minimum n relative to all normal transient waves excited in the channel has already passed.

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